## Domain: Number and Operations-Fractions

Student proficiency with fractions is essential to success in algebra at later grades. In grade five a critical area of instruction is developing fluency with addition and subtraction of fractions, including adding and subtracting fractions with unlike denominators. Students also build an understanding of multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

| Number and Operations Fractions | 5.NF |
|---|---|
| **Use equivalent fractions as a strategy to add and subtract fractions.**<br>1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.*<br>2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases with unlike denominators, e.g., by using visual fraction models or equations to represent the problems. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.* | |

In grade four students calculated sums of fractions with different denominators, where one denominator is a divisor of the other, so that only one fraction has to be changed. In grade five students extend work with fractions to add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions with like denominators **(5.NF.1▲).** (Adapted from Progressions 3-5 NF 2011)

Students find a common denominator by finding the product of both denominators. For $\frac{1}{3}+\frac{1}{6}$, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm. Student should first solve problems that require changing one of the fractions (as in grade four) and progress to changing both fractions. Students understand that multiplying the denominators will always give a common denominator but may not result in the smallest denominator; however, it is not necessary to find a least common denominator to calculate sums and differences of fractions.

To add or subtract fractions with unlike denominators, students need an understanding of how to create equivalent fractions with the same denominators before adding or subtracting, a concept learned in grade 4. In general they understand that for any whole numbers $a, b$ , and $n$ , $\frac{a}{b}=\frac{n \times a}{n \times b}$ (given that $n$ and $b$ are non-zero).

| **Examples:** |
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| $\frac{2}{5}+\frac{7}{8}=\frac{2}{5}\cdot\frac{8}{8}+\frac{7}{8}\cdot\frac{5}{5}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$ |
| $3\frac{1}{4}-\frac{1}{6}=3\frac{6}{24}-\frac{4}{24}=3\frac{2}{24}\text{ or }3\frac{1}{12}$ |

(Adapted from Progressions 3-5 NF 2011)

Students make sense of fractional quantities when solving word problems involving addition and subtraction of fractions referring to the same whole using a variety of strategies (**5.NF.2▲**).

| **Example:** Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 of a cup of sugar. How much sugar did he need to make both recipes? |
| --- |
| **Solutions:** |
| **Mental Estimation (MP.2):** A student may say that Jerry needs more than 1 cup of sugar but less than 2 |

cups, since both fractions are larger than 1/2, while at the same time both fractions are less than 1.

**Area Model to show equivalence (MP.5):** A student may choose to represent each partial cup of sugar using an area model, find equivalent fractions, and then add:

I see that 3/4 of a cup of sugar is equivalent to 9/12 of a cup, while 2/3 of a cup is equivalent to 8/12 of a cup. Altogether, I have 17/12 of a cup. This is more than one cup since $\frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}$.

$\frac{3}{4}$ cup of sugar

$\frac{2}{3}$ cup of sugar

(Adapted from Arizona 2012)

[Note: Sidebar]

**Focus, Coherence, and Rigor:**

When students meet standard **(5.NF.2▲),** they bring together the threads of fraction equivalence (learned in grades three through five) and addition and subtraction (learned in kindergarten through grade four) to fully extend addition and subtraction to fractions. (Adapted from PARCC 2012).

**Number and Operations—Fractions 5.NF**

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

3. Interpret a fraction as division of the numerator by the denominator (*a/b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

   a. Interpret the product *(a/b)* × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)× 4=8/3, and create a story context for this equation. Do the same with (2/3)× (4/5 ) = 8/15. (In general, (a/b)× (c/d) = ac/bd.)*

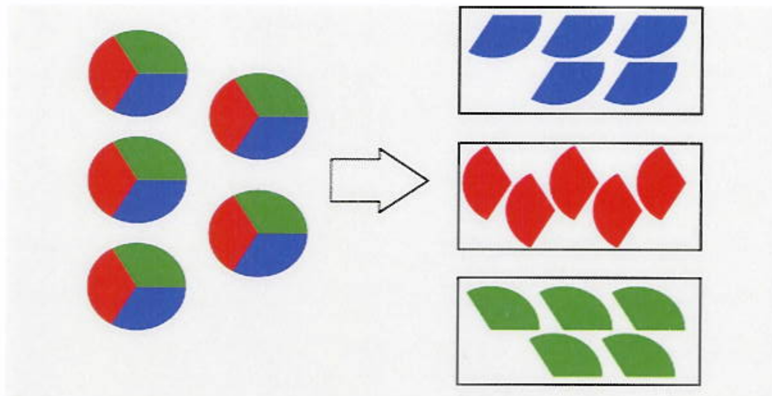
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

In grade five students connect fractions with division, understanding that 5 ÷ 3 = 5/3 or, more generally, *a/b* = *a* ÷ *b* for whole numbers *a* and *b*, with *b* not equal to zero

**(5.NF.3▲).** Students can explain this by working with their understanding of division as equal sharing.

**For example:** Sharing 5 objects equally among three shares, showing that

$5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$



If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute 1/3 of itself to each share. Thus each share consists if 5 pieces, each of which is 1/3 of an object, so each share is 5× 1/3 = 5/3 of an object. (Progressions 3-5 NF 2012)

Students solve related word problems and demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as “three-fifths” and, after experiences with sharing problems, students generalize that dividing 3 into 5 equal parts (3 ÷ 5 also written as $\frac{3}{5}$) results in the fraction $\frac{3}{5}$ (3 of 5 equal parts).

Students apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction **(5.NF.4▲).** Students multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately and solve problems in both contextual and non-contextual situations. Students reason about how to multiply fractions using fraction strips and number line diagrams. Using an understanding of multiplication by a fraction, students develop an understanding of a general formula for the product of two fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

| **Examples:** |
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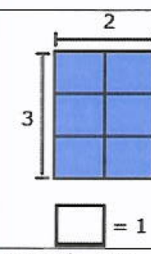
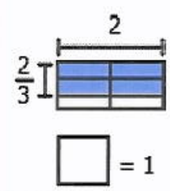
| When students multiply fractions such as in the problem $\frac{3}{5} \times 35$, they can think of the operation in more than one way:<br>• As $3 \times (35 \div 5)$, or $3 \times \frac{35}{5}$. (This is equivalent to $3 \times \left(\frac{1}{5} \times 35\right)$ and expresses the idea in standard **5.NF.4.b▲**).<br>• As (3 × 35) ÷ 5, or 105 ÷ 5. (This is equivalent to $\frac{105}{5}$.) |
| Students may be challenged to write a story problem for this operation.<br>"Mark's mother said he could have $\frac{3}{5}$ of the peanuts she bought for him and his younger brother to share. If she bought a bag of 35 peanuts, how many peanuts does Mark receive?" |

Building on previous understandings of multiplication, students find the area of a rectangle with fractional side lengths and represent fraction products as areas.

| **Examples of the reasoning called for in standard 5.NF.4.b▲.** | |
|---|---|
| Students have previously worked with examples of finding products as finding areas. In general, the factors in a multiplication problem represent the lengths of a rectangle and the product represents the area. | "This rectangle shows me that 2 × 3 = 6 by counting the number of square units and the side lengths of the rectangle."   |
| When students move to examples like $2 \times \frac{2}{3}$, they recognize that one side of the rectangle is less than a unit length (in this case; some might have mixed number side lengths). The idea of the picture is the same, but finding the area of the rectangle can be a little more challenging and requires reasoning about unit areas and how many parts unit areas are being divided into. | "I made a rectangle of sides 2 units and 2/3 of a unit. I can see that the two unit squares in the pictures are each divided into three equal parts (representing 1/3), with two shaded in each unit square (4 total). That means that altogether the area of the shaded rectangle is 4/3 square units."   |

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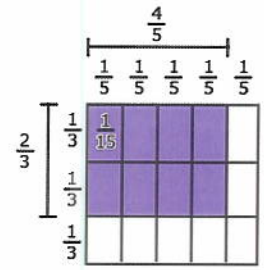
| Finally, when students move to examples like $\frac{2}{3} \times \frac{4}{5}$, they see that the division of the side lengths into fractional parts creates a division of the unit area into fractional parts as well. Students will discover that the fractional parts of the unit area are related to the denominators of the original fractions. Here, a 1×1 square is divided into thirds in one direction and fifths in another. This results in the unit square itself being divided into fifteenths. This reasoning shows why $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$. | "I created a unit square, divided it into fifths in one direction and thirds in the other. This allows me to shade a rectangle of dimensions 2/3 and 4/5. I noticed that 15 of the new little rectangles make up the entire unit square, so they must be fifteenths (1/15). Altogether, I had 2×4 of those fifteenths. So my answer is 8/15."   |

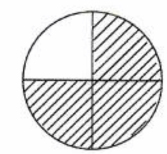
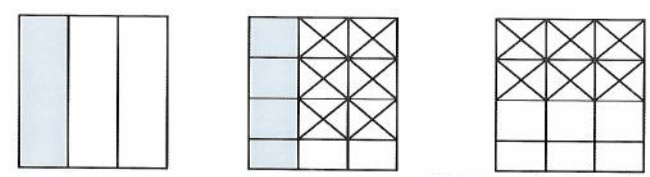
(Adapted from Arizona 2012)

[Note: Sidebar]

**Focus, Coherence, and Rigor:**
When students meet standard (**5.NF.4▲**), they fully extend multiplication to fractions, making division of fractions in grade six (**6.NS.1**) a near target.

Following is a sample classroom activity that connects the Standards for Mathematical Content and the Standards for Mathematical Practice.

## Connecting to the Standards for Mathematical Practice—Grade 5

| ***Standard(s) Addressed*** | ***Example(s) and Explanations*** |
|---|---|
| **5.NF.4:** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.<br>a. Interpret the product (*a*/*b*)×*q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3)×(4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*<br>b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be by multiplying the side lengths. Multiply fractional side lengths to find areas of<br>c. rectangles, and represent fraction products as rectangular areas. | **Task:** The following sequence of problems can be presented to students with tools provided (colored counters, rectangular and circular fraction pieces, fraction strips or rods, graph paper, etc.). Students should be encouraged to use their tools to solve the problem before presenting algorithms for the computations involved.<br>1. There are 18 marbles in a box. Two-thirds of the marbles are red. How many red marbles are there?<br>*Solution*: By seeing the 18 marbles as 3 sets of 6, we see that the solution is that 2×6 = 12 marbles are red. Notice that we found thirds of 18 first $\left(\frac{1}{3} \times 18 = 6\right)$ and then decided we wanted two-thirds. Several examples like this can used to show that $\left(\frac{a}{b}\right) \times q = a \times \left(\frac{q}{b}\right)$.<br>2. Roberto had $\frac{3}{4}$ of a pizza left. He gave $\frac{1}{3}$ of the leftover pizza to his kid sister. How much of the whole pizza did his sister get?<br>*Solution*: Using the same reasoning above, and using pictures to support the reasoning, we can see that one-third of three-fourths is one-fourth, so that Roberto's sister got $\frac{1}{4}$ of a whole pizza. <br>3. Mr. Jones was mowing his lawn and had $\frac{2}{3}$ of the lawn left to cut before he had to answer a phone call. After the call, he finished $\frac{3}{4}$ of what he had left. How much of the lawn did Mr. Jones cut after the phone call?<br>*Solution*: Here, we add the complication of finding fourths of thirds, which gives twelfths. After the phone call, Mr. Jones has cut six of those twelfths, so the answer is $\frac{6}{12} = \frac{1}{2}$ of the lawn. This can be illustrated with a rectangular fraction picture as shown. The lawn is first divided into thirds, one of which is shaded. Then the lawn is divided into fourths, and we notice that each of the small rectangular pieces represents $\frac{1}{12}$ of the entire lawn. Six of those are outlined in the picture.<br><br>Originally he mowed 1/3 of yard. He then mowed ¾ of what was left. How much of the total did he mow? Rearrange the 6 pieces and it is ½ of the area.<br><br>**Classroom Connections**<br>By building up students' understanding of fraction operations with different fraction models, a foundation can be laid |

for the algorithms to come. For example, eventually, students can attempt to justify the algorithm for multiplying fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ by understanding that first $\frac{c}{d}$ can be divided into $b$ equal parts; then, $a$ of those parts are taken. In total, $ac$ total parts of size $\frac{1}{bd}$ are taken.

**Connecting to the Standards for Mathematical Practice**
(MP.1) Students can be challenged to make sense out of each problem situation and to use their prior knowledge of fractions to try to model the situation and persevere in solving each problem.
(MP.4) Students are using fractional representations to model simple real-world situations. The real-world problems drive the mathematical concepts, as opposed to the opposite approach of learning algorithms and later applying them.
(MP.5) Students should have some familiarity with various fraction models and have the opportunity here to use them to solve actual problems and develop a conceptual understanding of fraction operations.

| Number and Operations—Fractions | 5.NF |
|---|---|

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n*× *a*)/(*n*× *b*) to the effect of multiplying *a*/*b* by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

In preparation for grade six work in ratios and proportional reasoning, students interpret multiplication as scaling (resizing) (**5.NF.5▲**) by examining how numbers change as they multiply by fractions. Students should have ample opportunities to examine the following cases: a) that when multiplying by a fraction greater than 1, the number increases, and b) that when multiplying by a fraction less the one, the number decreases. This is a new interpretation of multiplication and one that needs extensive discussion and explanation by students.

| **Examples:** |
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| "I know $\frac{3}{4} \times 7$ is less than 7, because I make 4 equal shares from 7 but I only take 3 of them (¾ is a fractional part less than one). If I'm taking a fractional part of 7 that is less than 1, the answer should be less than 7." |
| "I know that $2\frac{2}{3} \times 8$ should be more than 8, because 2 groups of 8 is 16 and $2\frac{2}{3} > 2.$ Also, I know the answer should be less than 24 = 3 × 8, since $2\frac{2}{3} < 3.$" |
| "I can show by equivalent fractions that $\frac{3}{4} = \frac{3 \times 5}{4 \times 5}$. But I also see that $\frac{5}{5} = 1$, so the result should still be equal to $\frac{3}{4}$." |

Students apply their understanding of multiplication of fractions and mixed numbers to solve real-world problems using visual models or equations **(5.NF.6▲)**.

| Number and Operations—Fractions 5.NF |
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| **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**<br>7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.<br>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*<br>b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × 1/5 = 4.*<br>c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* |

Students apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions **(5.NF.7▲)**, a new concept at fifth grade. Students will extend their learning about division of fractions in simpler cases here in grade five to the general case in grade six (division of a fraction by a fractions is not a requirement at this grade). Students use visual fractions models to show the quotient and solve related real-world problems.

| **Examples of the reasoning called for in standard (5.NF.7▲)** |
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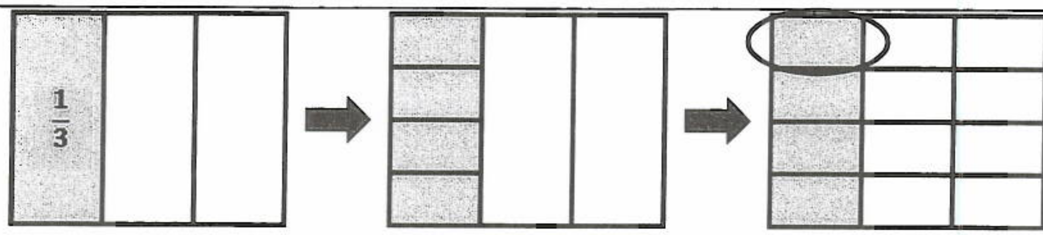
| **Partitive Division (Fair-Share Division) for dividing a unit fraction by a whole number:**<br>Four students sitting at a table were given 1/3 of a pan of brownies to share equally. What fraction of a pan of brownies will each student get if they share the pan of brownies equally?<br>**Solution:** The diagram shows the 1/3 of a pan of brownies divided into four equal shares. When replicated to fill out the entire pan, it becomes clear that each piece is 1/12 of an entire pan. (Indeed, if the 1/3-sized pieces are each in turn divided into 4 equal pieces, this makes a total of 12 equal pieces of the original whole.) |

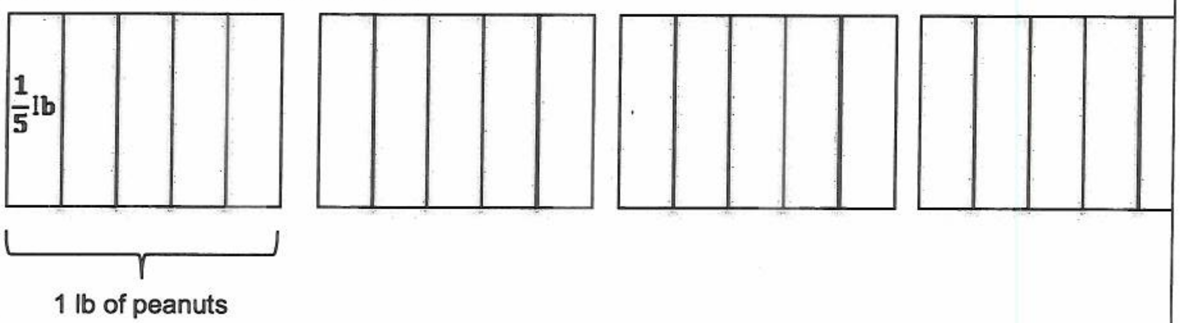



Students express their problem with an equation and relate it to their visual model: 1/3 ÷ 4 = 1/12, which is the same as ¼ x 1/3. (MP.2, MP.4)

**Quotitive Division (Measurement Division) for dividing a whole number by a unit fraction:**

Angelo has 4 pounds of peanuts. He wants to give each of his friends 1/5 of a pound. How many friends can receive 1/5 lb. of peanuts?

**Solution:** The question is asking how many groups of sized 1/5 lb. are found in 4 (whole) pounds? This leads us to draw 4 wholes, divide each of them into 1/5-lb. pieces, and count up how many of these pieces are shown.




We see that there are 20 (twenty) 1/5-lb sized portions in the original 4 pounds.

(Alternatively, a student may reason that since there are 5 (1/5-lb) size portions in each individual pound, and so there are 5 × 4 = 20 total. This reasoning lends itself to proportional reasoning in grades 6 and 7.)

(Adapted from Arizona 2012 and KATM 5th FlipBook 2012)